DESIGNING AND TESTING OF THE 5TH ORDER CHEBYSHEV LC-LPF USING VIRTUAL METHODS

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Abstract – *This paper presents a method for designing of a 5th order Chebyshev LC band-pass filter together with virtual testing using two programming and simulation environments used in engineering: LabVIEW and OrCAD. A good designing of a Chebysev LC band-pass filter assures an attenuation-frequency characteristic with an equal ripple variation in the passing band and has a monotonous increasing in the stop band.*

INTRODUCTION

A low pass filter (LPF) is a circuit that permits to a signal to pass through it if the signal has a frequency between zero and a passing limit frequency ω_t, with a maximum allowed attenuation a_t (fig. 1), while for the frequencies higher that a limit ω_b the signal is blocked with a minimum attenuation a_b. The frequency range between ω_t and ω_b is called transition band and for this domain, usually, there are no imposed restrictions for the attenuation value [1], [3].

The preset parameters for designing of a LPF are:

- the passing band, between the zero and a frequency limit ω_t;
- the blocking band, between ω_b and ∞;
- the maximum allowed value of attenuation in the passing band, $a_t\,[\mathrm{dB}]$;
- the minimum allowed value of attenuation in the stop band, $a_b\,[\mathrm{dB}]$;
- the load resistances at both ends of the filter, $R_1 = R_2 = R\,[\Omega]$, fig. 2.

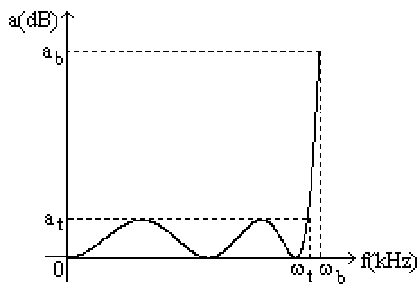

Fig. 1 – The attenuation-frequency characteristic for a Chebyshev LC-LPF

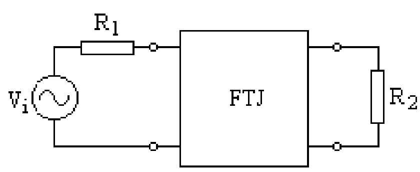

Fig. 2 – LPF closed at both ends with resistances

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1. DESIGNING OF THE CHEBYSHEV LPF

The filter that will be designed is a Chebyshev filter type, having the characteristics that the attenuation depending on frequency varies with equal ripple in the passing band (having equals minimum and maximum) and in the stop band the attenuation increases monotonous, as is shown in figure 1 [1], [3].

The first step is to normalize the value of the elements. To do that, the following unity values are chosen: $\omega_u = \omega_t$, $R_u = R$, and the elements L_u and C_u can be calculated as follow:

$$L_u = \frac{R_u}{\omega_u}; \quad C_u = \frac{1}{R_u \cdot \omega_u} \tag{1}$$

In the passing band, the reflexion coefficient ρ can be used instead the maximum allowed attenuation, which has the value:

$$\rho^2 = 1 - 10^{0.1 \cdot a_t} \tag{2}$$

To find the order of the Chebyshev LC-LPF, the following relation can be used:

$$n \geq \frac{a\cosh\left(\sqrt{\left(10^{0.1 \cdot a_b} - 1\right)\left(\rho^{-2} - 1\right)}\right)}{a\cosh\left(\omega_b\right)} \tag{3}$$

Choosing the variant of the circuit depends on the filter utilization. One possibility of hardware implementation is with inductors connected to the ground by capacitors like in fig. 3. This structure is best suitable especially at high frequency operation.

The normalized values of the inductances l_i and capacitances c_i are calculated using the following relations:

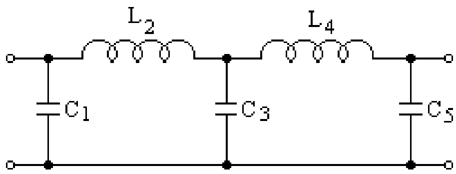

Fig. 3 – Schematic diagram of the Chebyshev LC-LPF

$$a_i = 2 \cdot \sin\frac{(2i-1)\cdot\pi}{2n}; \qquad b_i = \frac{b_0^2 + \sin^2\left[2(i-1)\cdot\frac{\pi}{2n}\right]}{b_{i-1}} \tag{4}$$

where:

$$b_0 = \sinh(p); \qquad p = \frac{1}{n} \cdot a\cosh\left(\frac{1}{\rho}\right) \tag{5}$$

The normalized values of the elements are: $\frac{a_i}{b_i}$, where for odd values of *i* are obtained the values of the capacitances c_1, c_3, c_5, and for even values of *i* are obtained the values of the inductances l_2 and l_4, (fig. 3).

The filter attenuation is calculated with the relation (6):

$$a(\omega) = 10 \cdot \log\left[1 + \varepsilon^2 \cdot T_n^2(\omega)\right] \tag{6}$$

where: $T_n(\omega)$ is the Chebyshev polynomial of order n, and:

$$\varepsilon^{-1} = \sqrt{\rho^{-2} - 1} \tag{7}$$

The Chebyshev polynomial can be expressed using trigonometric functions:

$$T_n(\omega) = \begin{cases} \cos\left(n \cdot ar\cos(\omega)\right), & \omega \leq 1 \\ \cosh\left(n \cdot ar\cosh(\omega)\right), & \omega \geq 1 \end{cases} \tag{8}$$

The nominal values (un-normalized) of the capacitances and inductances will be:

$$C_i = c_i \cdot C_u$$
$$L_i = l_i \cdot L_u \quad (9)$$

A. *For designing of a Chebyshev LC-LPF, it is starting with the following initial parameters:*

- passing band: $0L\ 10\,kHz$;
- the value of reflexion coefficient ρ admitted in the passing band: $\rho = 15\%$;
- blocking band: $16kHz L\ \infty$;
- the minimum allowed value of attenuation in the stop band;
- the load resistances at both ends of the filter: $R_1 = R_2 = 600\Omega$.

These imposed conditions for the Chebyshev LPF are depicted in fig. 4.

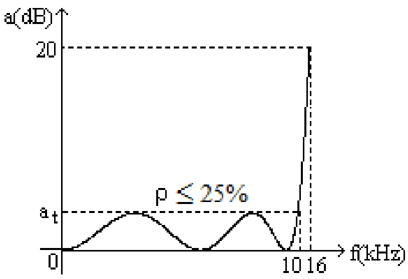

Fig. 4 – The imposed conditions for the Chebyshev LPF

By normalisation in respect to the limit passing frequency it is obtained:

$$\omega_b = \frac{f_b}{f_t} = 1.6 \quad (10)$$

And the filter order required is:

$$n \geq \frac{a\cosh\left(\sqrt{\left(10^{0.1 \cdot a_b} - 1\right)\left(\rho^{-2} - 1\right)}\right)}{a\cosh(\omega_b)} \geq \quad (11)$$
$$\geq 4.6576$$

We choose: $n = 5$.

The p and b_0 factors are then calculated:

$$p = \frac{1}{n} \cdot a\cosh\left(\frac{1}{\rho}\right) = 0.5169 \quad (12)$$

$$b_0 = \sinh(p) = 0.5402 \quad (13)$$

For the normalized values of the capacitors and inductors, using the relations (4), there are obtaining the following values:

i	$\frac{a_i}{b_i}$	
1	1.144	$c_1 = c_5$
2	1.3715	$l_2 = l_4$
3	1.9722	c_3

Because we choose a greater degree than released from calculus (because it was not an integer), the limit passing frequency will be calculated at the blocking attenuation equal to 20dB.

How:

$$a = \begin{cases} 10 \cdot \log\left[1 + \frac{\cos^2(n \cdot a\cos(\omega))}{\rho^{-2} - 1}\right], \omega \leq 1 \\ 10 \cdot \log\left[1 + \frac{\cosh^2(n \cdot a\cosh(\omega))}{\rho^{-2} - 1}\right], \omega \geq 1 \end{cases} \quad (14)$$

and:

$$a_b = 10 \cdot \log\left[1 + \frac{\cosh^2(n \cdot a\cosh(\omega_b))}{\rho^{-2} - 1}\right] = \quad (15)$$
$$= 20$$

It is obtained that:

$$\omega_b = \cosh\left(\frac{1}{n} \cdot ar\cosh\left(\sqrt{\frac{\left(10^{0.1 \cdot a_b} - 1\right) \cdot}{\left(\rho^{-2} - 1\right)}}\right)\right) \quad (16)$$

$$\omega_b = 1.5145$$

Because the imposed blocking frequency is $f_b = 16kHz$, the unity frequency is:

$$f_u = \frac{f_b}{\omega_b} = \frac{16}{1.5145} = 10.565\,kHz \quad (17)$$

results that the passing band is increased with 0.565 KHz.

The normalized values of the elements will have the following values:

$$L_u = \frac{R_u}{2\pi \cdot f_u} = 0.009H$$
$$C_u = \frac{1}{2\pi \cdot f_u \cdot R_u} = 25.108nF \qquad (18)$$

and the nominal values (un normalized) are:

$$C_{1a} = C_{5a} = c_1 \cdot C_u = 28.723nF$$
$$L_{2a} = L_{4a} = l_2 \cdot L_u = 12.4mH \qquad (19)$$
$$C_{3a} = c_3 \cdot C_u = 49.519nF$$

B. For designing of a Chebyshev LC-LPF, it is starting with the following initial parameters:

- passing band: $0L\ 10kHz$;
- the value of reflexion coefficient ρ admitted in the passing band: $\rho = 20\%$;
- blocking band: $16kHz L\ \infty$;
- the minimum allowed value of attenuation in the stop band;
- the load resistances at both ends of the filter: $R_1 = R_2 = 600\Omega$.

According to relations (11), (12) and (13), it was obtained the value for filter order:

$$n = 4.3742 \qquad (20)$$

and the following integer value was chosen: $n = 5$.

$$p = 0.4585,\ b_0 = 0.4747 \qquad (21)$$

The normalized values for the capacitances and inductances, according to (4), are the following:

i	$\frac{a_i}{b_i}$	
1	1.3019	$c_1 = c_5$
2	1.3456	$l_2 = l_4$
3	2.1286	c_3

The recalculated values for the passing frequency limit for which the blocking attenuation is 20dB are:

$$\omega_b = 1.4496$$
$$f_u = 11.073kHz \qquad (22)$$

Again, the passing band is increased with 1.073 KHz. The normalized values of the elements are:

$$L_u = 0.0087H$$
$$C_u = 24.033nF \qquad (23)$$

and the nominal values (un normalized) are:

$$C_{1b} = C_{5b} = c_1 \cdot C_u = 31.288nF$$
$$L_{2b} = L_{4b} = l_2 \cdot L_u = 11.6mH \qquad (24)$$
$$C_{3b} = c_3 \cdot C_u = 51.155nF$$

C. For designing of a Chebyshev LC-LPF, it is starting with the following initial parameters:

- passing band: $0L\ 10kHz$;
- the value of reflexion coefficient ρ admitted in the passing band: $\rho = 20\%$;
- blocking band: $16kHz L\ \infty$;
- the minimum allowed value of attenuation in the stop band;
- the load resistances at both ends of the filter: $R_1 = R_2 = 600\Omega$.

According to relations (11), (12) and (13), it was obtained the value for filter order:

$$n = 4.1497 \qquad (25)$$

and the following integer value was chosen: $n = 5$.

$$p = 0.4127,\ b_0 = 0.4245 \qquad (26)$$

Now, the normalized values for the capacitances and inductances, according to (4), are the following:

i	$\frac{a_i}{b_i}$	
1	1.4559	$c_1 = c_5$
2	1.3066	$l_2 = l_4$
3	2.2833	c_3

The recalculated values for the passing frequency limit for which the blocking attenuation is 20dB are:

$$\omega_b = 1.4019$$
$$f_u = 11.413\text{kHz} \qquad (27)$$

The passing band is increased with 1.073 KHz. The normalized values of the elements are:

$$L_u = 0.0084\text{H}$$
$$C_u = 23.241\text{nF} \qquad (28)$$

and the corresponding nominal values are:

$$C_{1c} = C_{5c} = c_1 \cdot C_u = 33.837\text{nF}$$
$$L_{2c} = L_{4c} = l_2 \cdot L_u = 10.9\text{mH} \qquad (29)$$
$$C_{3c} = c_3 \cdot C_u = 53.067\text{nF}$$

3. TESTING OF THE CHEBYSHEV LC-LPF USING THE LABVIEW PROGRAMMING ENVIRONMENT

For testing of the filters characteristics using the calculated values for the filter elements, a LabVIEW application was developed. The block diagram of the application is presented in fig. 5 and the front panel is showed in fig. 6.

The application traces the filter characteristics within the passing band and for the entire band at logarithmic scale and permit to choose the filter order and the three values for the reflexion coefficient in the passing band.

It is observed that the number of the maximum and minimum values is equal with the number of the components from the filter schematic diagram. Also, in the passing band, the attenuation-frequency characteristic varies with equal ripple and in the blocking (stop) band the attenuation increases monotonously.

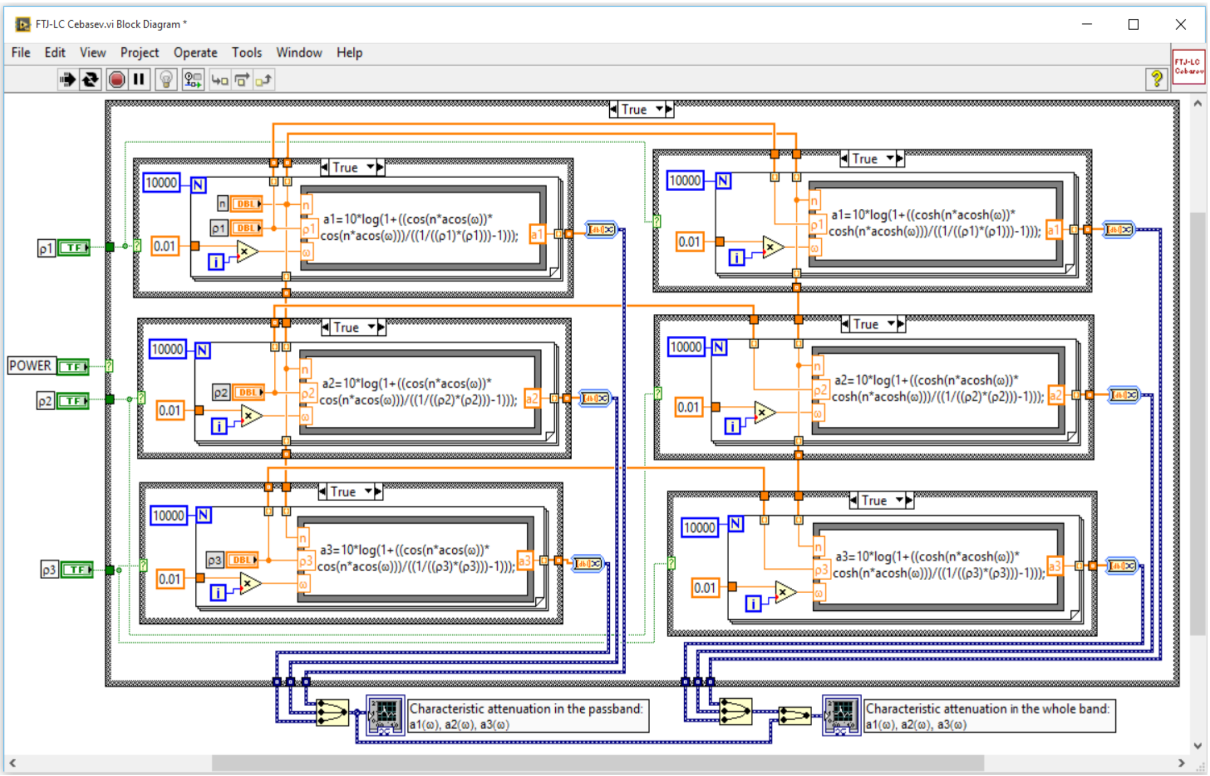

Fig. 5 – LabVIEW block diagram of the virtual instrument

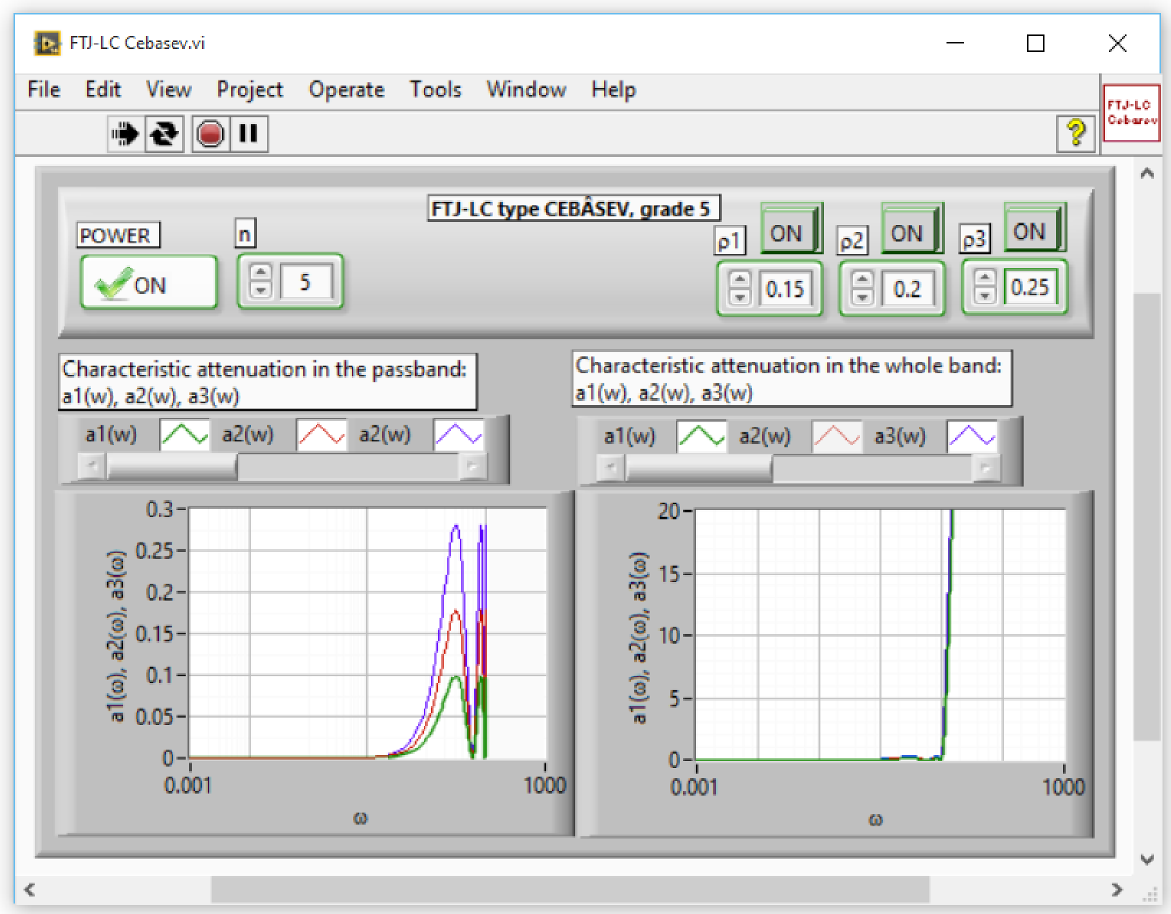

Fig. 6 – Frontal panel of the filter analysis and testing implemented in LabVIEW

4. TESTING THE CHEBYSEV LC- LPF USING ORCAD/PSPICE

The Chebyshev LC-LPF electrical circuits with the designed values for the three cases of the reflexion coefficient were simulated in PSPICE using OrCAD. The simulated electrical circuits with their characteristics are presented in fig. 7, 8 and 9.

It is observed that the attenuation-frequency characteristics are similar with the computed ones in LabVIEW. meaning that the entire design of the filter was correct.

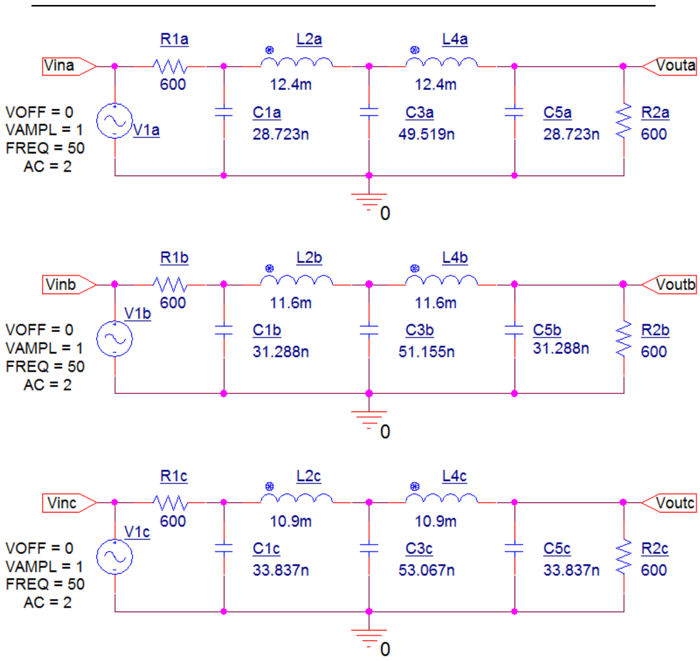

Fig. 7 – The Chebyshev LC-LPF simulated în OrCAD

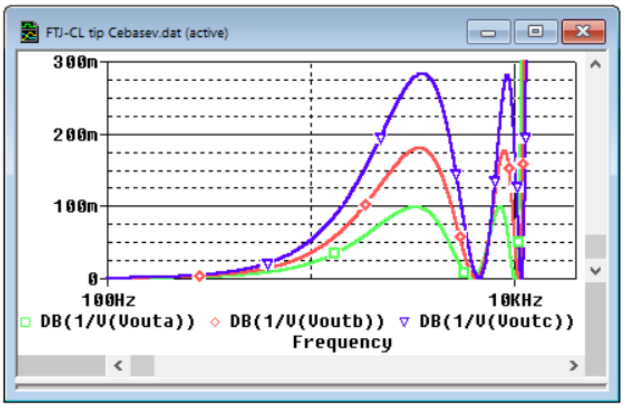

Fig. 8 – The attenuation–frequency characteristics in the passing band in OrCAD/PSPICE

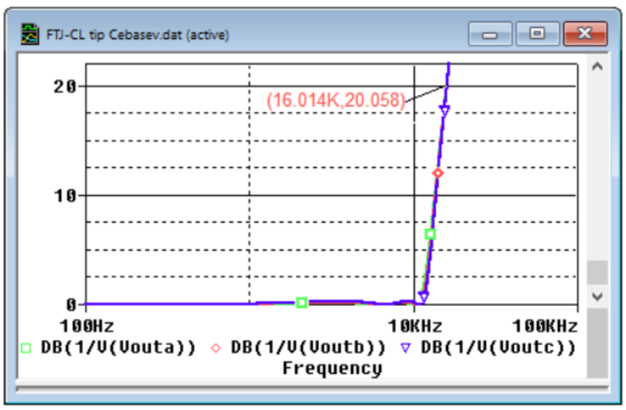

Fig. 9 – The attenuation–frequency characteristics over the entire band

5. CONCLUSIONS

- The virtual instrument implemented in LabVIEW can be used in the designing and analyze of the filters and permits justifying of the correct choice of the degree of the filters for the three values of the reflection coefficient using the attenuation characteristics;
- The correctness choice of the n degree for a filter permits the continuing of the calculus for determining the values required for LC components from Chebyshev LC-LPF filters;
- For the same degree n of a filter, increasing the reflection coefficient ρ (and also the value of the maximum allowed attenuation in the passing band) has as consequence the increasing of the passing band of filter, and for the same blocking frequency the same value for the minimum allowed attenuation is obtained.
- The two methods for virtual testing in the engineering environments LabVIEW and OrCAD have conducted to obtaining the similar results, which is a prove that the designing and testing methods are correct:

- In the passing band, the attenuation-frequency characteristic for a 5th Chebyshev LC-LPF has an equal variation of the ripple meaning that the approximation error has the equal maximum and minimum and the number of these is the same as the number of components from the filter structure;

- In the blocking band, the attenuation-frequency characteristic for a Chebyshev increases monotonously.

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